Cave and Karst Science, Vol.53, No.2, (2026) 81 – 84
Transactions of the British Cave Research Association

ISSN 1356-191X

Zero and pre-Darcy flow, and a brief update on speleogenesis research

Trevor FAULKNER
University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK.
Email: trevor.marblecaves@gmail.com

Abstract: Karst caves in sedimentary limestones commonly develop in a three-stage process of initiation, phreatic enlargement, and then vadose entrenchment. A recent paper about zero and pre-Darcy flow in low-permeability porous media raises some doubts about the assumed initial conditions for speleogenesis, with possible implications for the Palmer / Dreybrodt Model, the Inception Horizon Hypothesis and the speleogenesis of marble caves. Other recent clarifications of speleogenetic processes are also discussed.

Keywords: Fast Rate Law; marble; pre-breakthrough kinetics; tectonic inception

***Received**: 12 April 2026; **Accepted**: 05 May 2026.*

The Palmer / Dreybrodt Model (Palmer, 1981; Dreybrodt, 1990; Palmer, 1991; Klimchouk *et al*., 2000; Palmer, 2007) explained the physico-chemico dissolutional processes that create the triple porosity hydrogeology observed in karstic sedimentary limestones (Ford and Williams, 2007, p.104). The model was based on speleological research up to the 1990s that used a one-dimensional (1-D) setting. Karst cave passages commonly develop in three stages (Dreybrodt and Gabrovsek, 2000: Fig.10): laminar *pre-breakthrough* phreatic flow along an inception horizon (Lowe, 1992; Lowe and Gunn, 1997), a pore system, or a fracture; commonly turbulent *post-breakthrough* phreatic flow, when the conduit enlarges uniformly under aggressive conditions; and vadose entrenchment, after the passage has grown too large to remain water-filled, perhaps after local isostatic uplift. The breakthrough concept, based on a kinetic trigger idea by White (1977), overcame the difficulty realized by Weyl (1958) that dissolution in a narrow tube should become impossible beyond a short penetration length in water that was increasingly saturated by dissolved calcite. Instead, slow pre-breakthrough dissolution can continue to the exit, even in water that approaches 100% saturation, so that the whole conduit enlarges slowly whilst the calcite concentration gradually decreases as the flow rate increases. In the simplest case of an $H_2O{-}CO_2{-}CaCO_3$ phreatic system, without other acids being present, both the time to breakthrough and the pre- and post-breakthrough enlargement rates depend on geometry, temperature, and CO_2 and $CaCO_3$ concentrations in solution. At each point of a growing conduit, from Palmer (1991, Eq. 6), the solution rate of limestone dissolution,

$$S = 31.56k(1-C/C_s)^n/2.7 \text{ cm a}^{-1},$$

where the density of limestone is 2.7gm cm^{-3}, k mg-cm/litre-second is the reaction rate coefficient, C mg/litre is the concentration of dissolved calcite, C_s is the saturation concentration at the point temperature and P_{CO_2}, and n is the reaction rate order.

During the initial pre-breakthrough stage, Darcy's Law specifies a linear relationship between the volumetric rate and velocity of fluid flow and the hydraulic gradient (HG: head/path length) that drives the flow, when dissolution utilizes slow high-order kinetics with n>1. As explained by Palmer (1991: Fig.11), the *effective* saturation concentration (C_s') reduces from C_{so}, the saturation concentration in open conditions at the stream entry to the conduit, where CO_2 is resupplied from the surface or cave atmosphere and any organic decay, to C_{sc}, the saturation concentration in completely closed phreatic conditions with no available CO_2. The extreme ratios of C_{sc}/C_{so} vary from 16% at an initial P_{CO_2} of 0.02% atmosphere at 0°C to 73% at a P_{CO_2} of 10%, also at 0°C (Faulkner, 2022: Table 1). After breakthrough, when the penetration length reaches the exit, the walls of the conduit enlarge at a temperature- and P_{CO_2}-dependent high constant rate that utilizes fast first-order (linear) kinetics with n=1, provided that the flow rate maintains the calcite concentration below 55−85% saturation. CO_2 in solution is consumed as the limestone is dissolved as calcium bicarbonate along the conduit in increasingly closed conditions. That reduces C_s', which compensates for the reducing amount C of limestone dissolved downstream as P_{CO_2} reduces, so that the reaction rate k and the solution rate S remain constant along the length. Additionally, at each short length of the conduit, the new amount of limestone dissolved is balanced by the increase in volume, which ensures that C and therefore C_s', k and S remain constant in time. Surprisingly early in the evolution of modern karst studies, both Laptev (1939) and Bögli (1964) demonstrated that, if two streams meet with differing amounts of dissolved carbon dioxide and/or calcium carbonate, the mixed-water output is more aggressive than either input, even if both are nearly saturated. This arises from the non-linear relationships of $CaCO_3$ solubility with P_{CO_2} and temperature, whereas mixing is a linear process. Thus, *mixing corrosion* (MC) commonly increases the dissolution rate downstream of a junction of two conduits.

Based on modelling the chemical and flow kinetics of calcite dissolution and by simulating previous practical experiments, Groves and Howard (1994) proposed that there must be threshold sizes below which the initial karstification of tiny flow routes is impossible within any realistic geologic timescale. For example, their Figures 5 and 6 claimed to show that 1000m-long conduits with an initial diameter of 0.25mm and with entrance atmospheric levels of dissolved CO_2 would experience no passage enlargement at HG $< c.$ 1.5%. However, Dreybrodt (1996) reported that the Groves and Howard (1994) finite-difference numerical model used length increments that were too large. In contrast, he concluded that there are no threshold widths below which the karstification of narrow broad fractures and circular conduits is impossible. However, Dreybrodt and Gabrovsek (2000, p.190) then accepted that breakthrough time needs to be less than the age of the limestone.

Dreybrodt and Gabrovsek (2000) also concluded that if the inflow to an aperture is already greater than 99% saturated, then breakthrough does not occur. Instead, the conduit widens uniformly along its entire length, but at a rate of only 10^{-9}cm per year. It was observed by Faulkner (2006a) that similar conditions are created for sub-micron-sized flow routes several kilometres in length with negligible HG, even if the recharge contains no dissolved calcite, thus underpinning the Inception Horizon Hypothesis (Lowe, 1992; Lowe and Gunn, 1997). His paper also extended the Palmer / Dreybrodt Model to consider maximum rates and dissolution in pure cold water, even without any CO_2, and Faulkner (2006b) proposed that tectonic inception can explain speleogenesis in indurated sedimentary limestones and in metamorphic limestones (marbles). For example, fractures created in response to isostatic uplift during the deglaciation of previously glaciated areas can transmit water in turbulent flow from overlying ice-dammed lakes with little dissolved calcite immediately. That would bypass the pre-breakthrough stage of speleogenesis and could create phreatic passages up to 2m wide (as observed in Norway) if the flow lasted for 1000 years, even at ~0°C with minimum P_{CO_2}.

The subject of speleogenetic dissolution has advanced further in the last 20 years, including in 2-D and 3-D sedimentary settings that are beyond the scope of this Report. A *Fast Rate Law* with linear kinetics at much higher than post-breakthrough reaction rates, as anticipated implicitly by Palmer (1991: Fig.8), was confirmed by theoretical predictions and by open system experiments at 10°C, 20°C and 25°C by Kaufmann and Dreybrodt (2007), as discussed by Faulkner (2022). They ran water down an inclined plate of limestone under open atmospheric conditions and also at P_{CO_2} = 1% and found that the Fast Rate Law applies for flow rates at <20% saturation, even with a low P_{CO_2}. Thus, there is a second kinetic trigger point that had not previously been identified, although it has not yet been confirmed experimentally for closed phreatic systems.

Data given by Palmer (1991: Table 1) had shown the variation of reaction coefficient (k) with temperature and P_{CO_2} that controls post-breakthrough limestone dissolution, to an accuracy of ±10−20%. That information was plotted by Faulkner (2006a: Fig.4) with interpolations and extrapolations to illustrate reaction coefficients at breakthrough from 0−25°C at P_{CO_2} from 0.003−100%. Considering here the constant maximum post-breakthrough rates of dissolution along conduits in continuously flowing phreatic water, initially with zero dissolved calcite in open conditions at the entrance:

$$S_{max} = 315.6k/2.7 \text{ mm a}^{-1},$$

from Palmer (1991: Eq. 6), as above.

These rates are plotted in Figure 1, across similar temperature and P_{CO_2} ranges, by fine-tuning post-breakthrough values of k, whilst keeping well-within the Palmer (1991) level of accuracy. Although the maximum dissolution rate always increases with temperature, it is almost independent of P_{CO_2} if P_{CO_2} <0.3%. That includes for the *c.* 0.03−0.04% minimum level of dissolved CO_2 under recent and present atmospheric conditions. It demonstrates that calcite can dissolve in water that contains zero CO_2, although it then saturates at only 10−13 mg litre^{-1} from 0−30°C (Faulkner, 2022: Table 1). It is seen that S_{max} remains significant at >0.6mm a^{-1}, even at 0°C with low P_{CO_2}. The constancy of the reaction coefficient for P_{CO_2} <0.3% was not apparent from Faulkner (2006a: Fig.4). The values of k used to construct Figure 1 are much less than the equivalent values calculated for the Fast Rate Law in open systems by Kaufmann and Dreybrodt (2007: Table 4). Hence, the maximum dissolution rates shown in Figure 1, which also do not include mechanical erosion, and potentially the rate at 0°C during deglacial speleogenesis when the water is not frozen in summer, should be even higher. The saturation levels below which breakthrough occurs (Palmer, 1991: Table 1) remain constant for P_{CO_2} <3% at 55%, 60%, 65%, 70%, 75%, 80% and 85% at each 5°C rising increment of temperature from 0−30°C (Faulkner, 2022: Fig.3).

A previous misunderstanding that the transition to post-breakthrough flow in limestone under hydraulic *(constant head)* control occurs commonly at a conduit exit size of *c.* 1cm was discussed by Faulkner (2013). The transition was assumed to coincide with the change from a wholly laminar flow to the onset of transitional turbulent flow at the Reynolds Number (R_e) of 2200 that applies in some artificial pipes. However, the conduit size for the onset of turbulence varies with HG, whereas that for the onset of first-order kinetics varies with the hydraulic ratio, which is HG divided by the path length (Head/Length^2). Thus, there is no typical R_e at the chemical breakthrough point. For tube lengths shorter than 452m at 10°C with R_e = 2200, breakthrough occurs first and transitional turbulence is soon reached under first-order kinetics. For longer lengths, transitional turbulence occurs first, which reduces the flow velocity, and chemical breakthrough can be delayed significantly. Thus, whether conduit length is less or more than the coincidental length is important for speleogenesis, because simpler models become increasingly inaccurate for longer lengths at low hydraulic gradients. Conduit exit diameters for breakthrough and transitional turbulence can be less than 1mm for short and steep conduits, with R_e<10 at breakthrough, but more than a metre for long and shallow conduits, with R_e>50,000 at breakthrough, showing that there are no real limits on exit sizes. Indeed, hydraulic ratios can be considered across 17 orders of magnitude. Based on the calculations utilized by Faulkner (2013), additional to that paper it can be shown that the coincidence lengths L_f and L_t for planar fissures and cylindrical tubes are independent of HG and that:

$$L_f = 10R_e\mu \text{ and } L_t = 5\pi R_e\mu,$$

where the dynamic viscosity of water, μ = 0.01793 and 0.01307 gm cm^{-1}s^{-1} at 0°C and 10°C, assuming that the density of water is 1gm cm^{-3}, and ignoring any variations in friction factor with aperture.

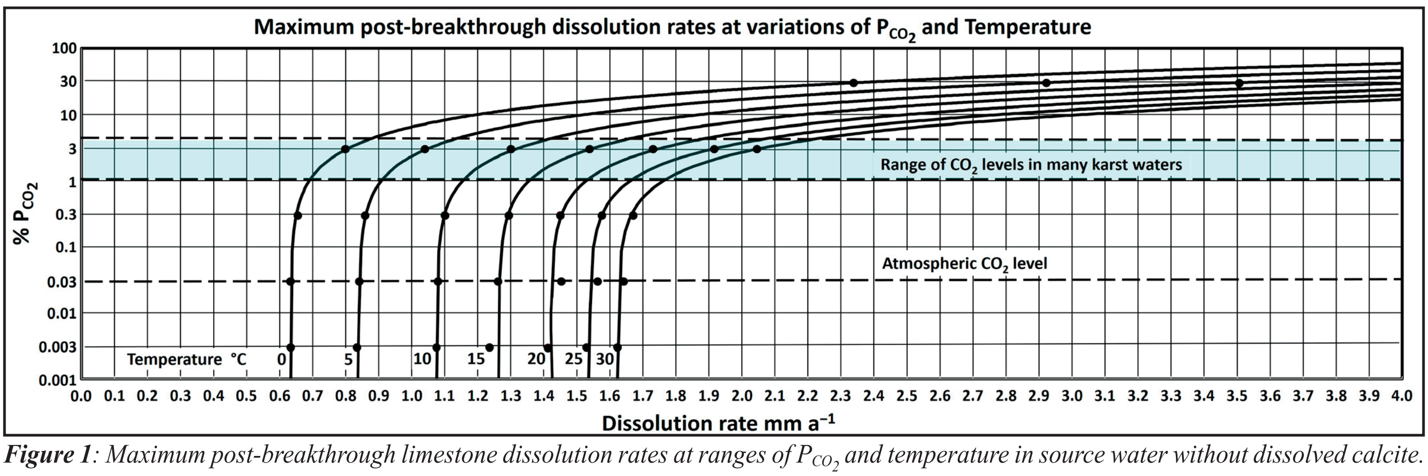

Figure 1: *Maximum post-breakthrough limestone dissolution rates at ranges of P_{CO_2} and temperature in source water without dissolved calcite.*

Hence, at temperatures of 0°C and 10°C, for an extremely small R_e of 10 for short conduits:

$$L_f = 1.8\text{m and } 1.3\text{m}$$
$$\text{and} \quad L_t = 2.8\text{m and } 2.1\text{m}$$

Similarly, for an extremely large R_e of 50,000 for long conduits:

$$L_f = c.\ 9000\text{m and } c.\ 6500\text{m}$$
$$\text{and} \quad L_t = c.\ 14{,}000\text{m and } c.\ 10{,}300\text{m}$$

Another interesting research question involves the relative importance of chemical dissolution and mechanical erosion during each stage of speleogenesis. As reported by Palmer (2007, p.148), wall scallops can be prevented from forming [and therefore also probably removed] by flow velocities $>3\text{ms}^{-1}$, especially if abraded by entrained sediment. Thus, with increasing flow velocity, scallops disappear at <1cm length. Passage enlargement then continues under *dominant mechanical erosion*, as discussed by Faulkner (2013), who deduced that this requires relatively steep conduits and large flow rates under a wide range of R_e values. The smallest radius at which unseen scallops could form is also *c.* 1cm. The presence of scallops on cave walls probably indicates that mechanical erosion was negligible when they formed. It was deduced by Faulkner (2013) that scallops form only in karst cave passages that are subject to post-breakthrough dissolution. He also considered the possibility that dominant mechanical erosion could occur before the onset of full turbulence in short steep fissures or tubes with HG >0.1−0.4. In most natural karst conduits, however, chemical breakthrough occurs first. It was also shown by Faulkner (2013) that cylindrical tubes under *constant phreatic recharge* can reverse back towards laminar flow with high-order dissolution kinetics via the *limiting recharge point* whilst HG and flow velocity reduce as size increases.

As was pointed out by Covington *et al.* (2015), existing models of cave passage speleogenesis relied solely upon the role of dissolution, whereas contemporary models of surface stream incision were based on stream power and the assumption that dissolution can be ignored. Their data from surface streams on limestone show that the effect of dissolution is near-constant at *c.* 1mm per year. In contrast, rates of mechanical erosion are probably highly variable, being more dependent upon factors related to extreme flood events. So far, the practical interplay of chemical dissolution and mechanical erosion in river channels has been studied largely in silicate lithologies. More recently, Dingle *et al.* (2025) reported that rates of dissolution and abrasion are broadly equivalent within two limestone bedrock channels in the northern English Pennines. Rapid formation of rockmill potholes in marble solely by the effects of mechanical erosion during deglacial jökulhlaups is being considered separately by this author. Thus, it seems that the relative contributions of chemical dissolution and mechanical erosion to the removal of limestone can vary from 0−100% in different karst settings and across a range of timescales.

Recent studies explore MC and speleogenetic inception within sedimentary limestones exhibiting high matrix and/or fracture permeability, using 2-D or 3-D models. A one-dimensional approach remains important, however, for investigation of karstification along extended flow routes in indurated limestones such as marbles, which initially possess negligible primary porosity and can display low fracture density. In such situations, early MC might apply. Dissolution and any later-stage MC after phreatic conduits reach maturity is also essentially a 1-D process, because other permeabilities are by then insignificant.

These ideas were explored by Faulkner (2022), who also confirmed the intriguing White (1977) Conjecture. This is that the mixing of two input streams with dissimilar but near-saturated chemistries could initiate fast first-order kinetics dissolution in the output stream immediately. However, it could not cause the Fast Rate Law regime to be entered.

Importantly, Teng *et al.* (2024) have reviewed many studies that show that flow velocity can exhibit a non-linear dependence on HG in low-permeability porous media, such as clay, shale and tight sandstones. Even more important, there can be zero flow if HG is below a threshold limit that depends on permeability. At each value of permeability, increasing HG above the threshold would allow low-velocity non-Darcy flow or *pre-Darcy flow* to increase, until Darcy flow occurs with a linear relationship at a higher HG (Teng *et al*., 2024: Fig.1). That paper provides a comprehensive review of the theories, experimental data, and modelling methods for such zero and pre-Darcy flows. Their Fig.3 is a log-log plot of threshold HG against permeability from challenging laboratory experiments, commonly at 20−30°C, where stated. Although there is some lowering of the threshold HG at higher temperatures as water becomes less viscous, the resulting fitted straight-line graph is a remarkably good fit for flows through various types of lithology ($R^2 = 0.92$), as reproduced here in Figure 2. The higher values of HG at which the Darcy Law is obeyed fully are less clear but seem to be only slightly above the threshold values. Although zero and pre-Darcy flows were not discussed for limestones, they could apply along inception horizons in other lithologies that are adjacent to limestone, and probably in porous and fractured limestone itself. Hence, speleogenetic inception could be prevented below the Figure 2 plot at both extremes of low gradients in extensive limestone aquifers and of ultra-high gradients >1, where lakes or dammed reservoirs lie above limestone bedrock. Dissolution is hardly relevant in relatively short geological timescales for silicate-based lithologies, whose permeability, and of course the HG, remain almost constant. When water can only just flow in or adjacent to limestone, however, dissolution will start to dissolve the limestone and increase the permeability, thereby advancing the onset of full Darcy flow. For such karst systems with low hydraulic gradients that are only just above the zero-flow threshold, the lower pre-Darcy flow velocity would necessarily delay breakthrough to fast dissolution kinetics.

Considering the concept of zero and pre-Darcy flow re-opens the debate initiated by Groves and Howard (1994) about threshold sizes for karstification, albeit in response to different reasoning, and this could have important implications. Permeability ranges of the relevant limestone lithologies, provided by Ford and Williams (2007: Fig.5.8), are included in Figure 2. These suggest that water flow and karstification via pre-breakthrough kinetics are commonly possible only in high- to low-grade marbles at a HG of 100,000 to 100 or in "limestone" at a HG of 50 to 2.

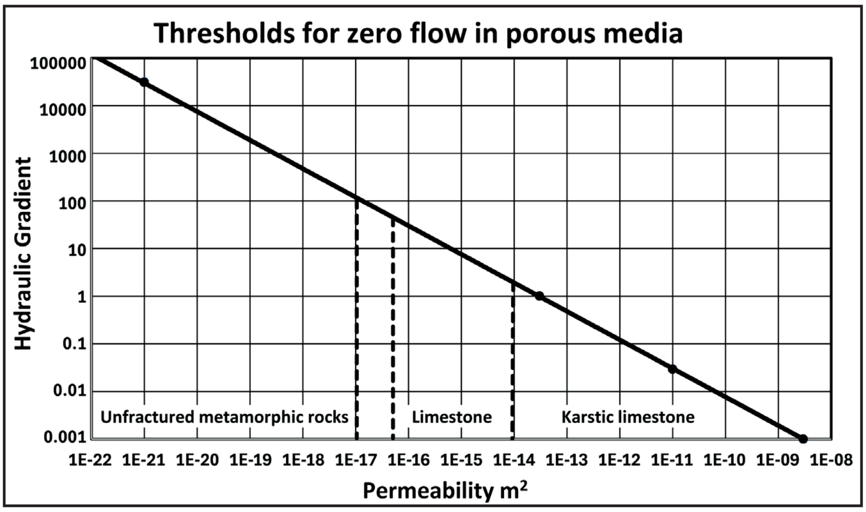

Figure 2 (right): *Thresholds for zero flow in porous media, based on Teng* et al. *(2024: Fig.3), with slight extrapolations. Lithology permeability ranges are from Ford and Williams (2007: Fig.5.8).*

Thus, although there are thousands of marble caves in Norway and many more elsewhere, apparently none of them should exist at all! Even considering that most karst sites in Norway were covered by deglacial ice-dammed lakes with depths ranging up to or beyond 150m, few of their potential flow routes could have experienced a HG >100 that would have initiated speleogenesis. The answer to this conundrum is that major isostatic uplifts that were triggered by deglaciation led to the rapid creation of neo-tectonic fractures large enough to transmit water immediately from the ice-dammed lakes at high speed during tectonic inception.

Such flows bypassed the pre-breakthrough kinetics stage and instantly utilized post-breakthrough dissolution kinetics or the *Fast Rate Law* conditions of Kaufmann and Dreybrodt (2007), as discussed above. Reversing this argument, it should be safe to build reservoirs in tectonically stable areas without fractures above limestones that have confirmed permeabilities $<10^{-15}m^2$ and above all marbles, depending upon the depth of water and the potential HG. If reservoirs *are* built in karst areas with fractures or a HG exceeding the local permeability threshold, karstification would rapidly reach post-breakthrough kinetics (Dreybrodt, 1996). The dissolution rate could then increase to that of the *Fast Rate Law*, leading to a rapid underground draining of the reservoir. Returning to Figure 2, for "karstic limestone", eventual karstification is possible for a HG >2 to $>8\times10^{-3}$, for which the lower limit is less than for the setting of many caves on Earth. Thus, the speleogenesis of nearly all caves in sedimentary limestone should be unaffected by the concept of zero and pre-Darcy flow.

To conclude, the basic 1-D physico−chemico models developed in the 1990s to explain speleogenesis in karst limestones remain entirely sound. Subsequent refinements include: extensions to 2-D and 3-D; the *Fast Rate Law*, which helps explain deglacial speleogenesis and tectonic inception; the non-equivalence of chemical breakthrough and onset of turbulence; a possible chemical reversal back to high-order kinetics; the confirmation of the White (1977) Conjecture; and early research on the effects of mechanical erosion. The concept of zero flow unless the hydraulic gradient exceeds a threshold for the local permeability provides strong support *in principle* for the Groves and Howard (1994) proposal. New practical laboratory experiments to determine thresholds for various types of limestone are therefore required. Although this concept is probably not relevant for most sedimentary limestones, it has major implications for speleogenesis in marbles, providing further support that most marble caves originated by way of tectonic inception.

Acknowledgements

The author thanks an anonymous and knowledgeable reviewer for providing helpful and valuable suggestions, and David Lowe for making Editorial improvements.

References

Bögli, A, 1964. Mischungskorrosion – ein Beiträg zur Verkarstungsproblem. *Erdkunde*, Vol.18, 83–92. [English translation by E Elder. 1985. Mixture Corrosion – a contribution to the karstification problem. *Cave Geology*, Vol.1(10), 393–406.]

Covington, M D, Gulley, J D, and Gabrovsek, F, 2015. Natural variations in calcite dissolution rates in streams: Controls, implications and questions. *Geophysical Research Letters,* Vol.42(8), 2836–2843.

Dingle, E H, Baynes, E R C, Hall, A, and Warburton, J, 2025. Erosion dynamics in carbonate bedrock channels inhibit weathering processes. *Earth Surface Processes and Landforms*, Vol.50(5), 18pp.

Dreybrodt, W, 1990. The role of dissolution kinetics in the development of karst aquifers in limestone: a model simulation of karst evolution. *Journal of Geology*, Vol.98(5), 639–655.

Dreybrodt, W, 1996. Principles of early development of karst conduits under natural and man-made conditions revealed by mathematical analysis of numerical models. *Water Resources Research*, Vol.32(9) 2923–2935.

Dreybrodt, W and Gabrovsek, F, 2000. Dynamics of the evolution of single karst conduits. 184–193 *in* Klimchouk, A B, Ford, D C, Palmer, A N, and Dreybrodt, W (eds), *Speleogenesis: Evolution of karst aquifers*. [Huntsville, Alabama, USA: National Speleological Society Inc.]

Faulkner, T, 2006a. Limestone dissolution in phreatic conditions at maximum rates and in pure, cold, water. *Cave and Karst Science*, Vol.33(1), 11–20.

Faulkner, T, 2006b. Tectonic inception in Caledonide marbles. *Acta Carsologica*, Vol.35(1), 7–21.

Faulkner, T, 2013. Speleogenesis and scallop formation and demise under hydraulic control and other recharge regimes. *Cave and Karst Science*, Vol.40(3), 113–132.

Faulkner, T, 2022. The role of mixing corrosion at different temperatures in the one-dimensional epigenic speleogenesis of indurated limestones, and the White (1977) conjecture. *Cave and Karst Science*, Vol.49(1), 19–34.

Ford, D and Williams, P, 2007. *Karst Hydrogeology and Geomorphology*. [Chichester, UK: John Wiley and Sons Ltd.] 562pp.

Groves, C G and Howard, A D, 1994. Minimum hydrochemical conditions allowing limestone cave development. *Water Resources Research*, Vol.30(3), 607–615.

Kaufmann, G and Dreybrodt, W, 2007. Calcite dissolution kinetics in the system $CaCO_3$–H_2O–CO_2 at high undersaturation. *Geochimica et Cosmochimica Acta*, Vol.71, 1398–1410.

Klimchouk, A B, Ford, D C, Palmer, A N, and Dreybrodt, W (eds), 2000. *Speleogenesis: Evolution of karst aquifers*. [Huntsville, Alabama, USA: National Speleological Society, Inc.] 527pp.

Laptev, F F, 1939. Aggressive action of water on carbonate rocks, gypsum and concrete. *Trudy Spetsgeo*, Vol.1. [Moscow–Leningrad: GONTI.] 120pp. [In Russian.]

Lowe, D J, 1992. The origin of limestone caverns: an inception horizon hypothesis. PhD Thesis. Manchester Metropolitan University.

Lowe, D J and Gunn, J, 1997. Carbonate Speleogenesis: An Inception Horizon Hypothesis. *Acta Carsologica*, Vol.XXVI/2, 457–488.

Palmer, A N, 1981. Hydrochemical factors in the origin of limestone caves. 120–122 *in* Beck, B F (Ed.), *Proceedings of the Eighth International Congress of Speleology*. [Bowling Green, Kentucky, USA. July 18 to July 24, 1981.]

Palmer, A N, 1991. Origin and morphology of limestone caves. *Geological Society of America Bulletin*, Vol.103, 1–21.

Palmer, A N, 2007. *Cave Geology*. [Dayton, Ohio, USA: Cave Books.] 454pp.

Teng, Y, Li, Z and, Chen, C, 2024. Review: Pre-Darcy flows in low-permeability porous media. *Hydrogeology Journal*, Vol.32, 1957–1977.

Weyl, P K, 1958. The solution kinetics of calcite. *Journal of Geology*, Vol.66, 163–176.

White, W B, 1977. Role of solution kinetics in the development of karst aquifers. 503–517 *in* Tolsen, J S and Doyle, F L (eds), *Karst Hydrology*. [Huntsville, Alabama, USA: UAH Press.]