The Parallel-Fed Voltage Multiplier

Voltage doublers and the Cockcroft-Walton voltage multiplier are examples of devices that use capacitors as charge-pumps to generate a high voltage at a low power in a relatively simple circuit. The conventional voltage multiplier is series-fed, acting as a 'bucket brigade' to pass the charge from one element to the next. This is difficult to analyse, as well as being inefficient, with a law of diminishing returns applying. A better scheme is the less well-known parallel-fed device, which has several advantages. ***David Gibson*** *analyses this device, arguing that it deserves to be better known, and suggests some possible cave electronics applications.*

The Cockcroft-Walton voltage multiplier, with its characteristic 'zig-zag' pattern of diodes (*Figure 1*), presents a familiar circuit to electronic engineers. It was invented in 1932 and has seen many applications in the generation of EHT at a relatively low power, such as in X-ray machines, CRT TVs, laser printers and ionisers. For these applications, a conventional switched-mode power supply (SMPS) would be expensive due to the high voltage requirement.

But even at lower voltages, the use of a SMPS can be more costly than a capacitor pump when only a small amount of power is required. Low voltage applications range from the modest voltage doublers used to generate the power rails needed for RS232 communications, through to 100–200 V generators for transcutaneous electrical nerve stimulation (TENS), which is used for pain relief and massage therapy.

The standard Cockcroft-Walton circuit is shown in *Figure 1*. The analysis and design of this circuit is not trivial, because the voltage and current waveforms are different for each stage. It is known that, optimally, the capacitors should increase in value from C at the input to NC at the output. Another disadvantage is that the output voltage is diminished by a quantity proportional to the square of the number of diodes present. Thus, when the number of stages reaches a critical value, further stages do not contribute any additional output.

The ***parallel-fed voltage multiplier*** (PFVM), is an improvement to the standard C&W design that was described in Wireless World in 1984 (Purves and Prescott, 1984). It is shown in *Figure 2*.

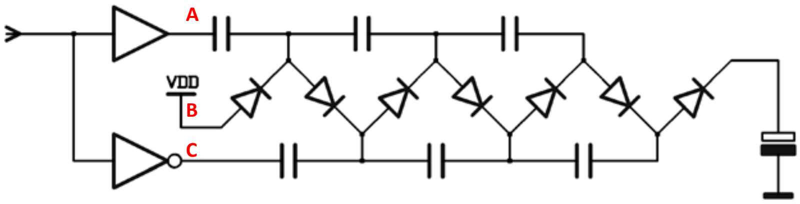

Figure 1 – The Cockcroft-Walton Voltage Multiplier
This configuration is for a unipolar drive from digital logic. It can also be used by grounding points B and C and applying a bipolar (i.e. AC) waveform to point A.

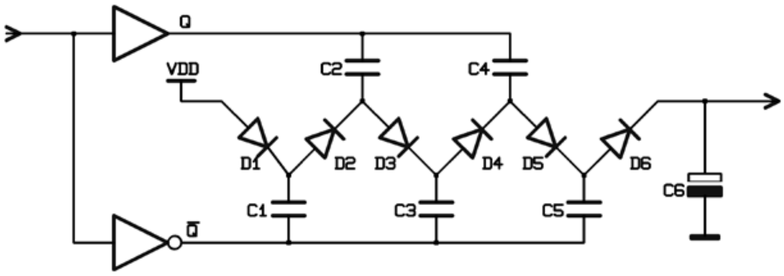

Figure 2 – The Parallel-Fed Voltage Multiplier
This is driven from a unipolar square wave. Six stages are shown here, and the output is therefore six times the input voltage, less six diode drops.

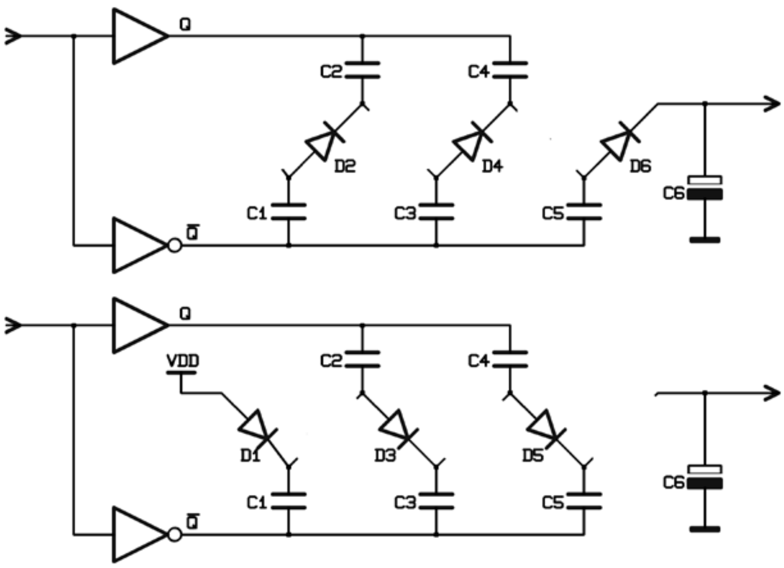

Figure 3 – The Two Phases of the PFVM
Top (a): *the input is LO; the top driver sinks current.*
Bottom (b): *the input is HI; the bottom driver sinks current.*

The salient feature is that the N capacitors are driven in parallel. They are all the same value, and the contribution to each stage of the multiplier is simply the driving voltage, less one diode drop. The output is therefore $N(U_S - U_D)$ and can be increased indefinitely, simply by adding additional stages[1]. There is, however, a drawback, in that the required voltage rating of the components must increase with each stage, which is a difficulty if the purpose of the charge-pump is to generate an EHT voltage. Nevertheless, this is an interesting device that deserves to be more widely known. For applications such as the one described by Purvis and Prescott, which was a battery-operated electrical muscle stimulator, the PFVM has distinct advantages over the C&W design.

1 Symbols and their meanings are listed at the end of this article, on page 22

I will now analyse this device, giving its performance in terms of its equivalent output resistance and power efficiency. The underlying point is that if we know how it works, we better know how to use it. For example, it might prove to be useful as a driver for LED cap-lamps, but we need to derive some design parameters first.

How It Works

The device has a symmetry about it that the C&W circuit does not, so a description of its operation is more straightforward. To describe it in detail is still, however, a wordy exercise so I will leave this for the reader, aided by *Figure 3*, which shows the two phases of operation.

In summary, when the pump has reached a stable operating point, the capacitors are charged (reading left to right in *Figure 2*) to one to six times U_S. Picking a capacitor in the middle of the chain, say C3 in *Figure 3a*, we assume that it is charged to nearly 3U. The Q driver then goes LO, and C3 is bumped up to $4U_S$, when it is able to discharge by an amount ΔU into C4, which thereby rises to nearly $4U_S$. Because the capacitors have the same value, each will transfer, in a half-cycle of discharge, an amount of charge Q given by

$$Q = C\,\Delta U = \int_0^{\tau/2} I\,dt \qquad (1)$$

There are a couple of points to note. Firstly, $\Delta U << U_S$. The capacitors only transfer a small amount of their total charge and the operating point for each capacitor is close to NU. Secondly, although the output voltage under a no-load condition will approach NU_S, it will be lower if there is a finite load. There are also the diode voltage drops to consider, but note that it would be wrong to assume that the voltage drop is a standard 0.6 V. In fact, despite what you might have been taught at college, there is no such thing as a diode 'knee' voltage[2]. The diode's voltage drop is a logarithmic function of current, and if you operate the charge pump at a very low current, the effective voltage drop across each diode will be noticeably less than 0.6 V.

Since each stage of the pump is transferring to the next stage a charge Q in the cycle time τ, it follows that the mean load current must be

$$I_L = \frac{Q}{\tau} = C\frac{\Delta U}{\tau} \qquad (2)$$

and that the mean source current must be

$$I_S = N I_L \qquad (3)$$

To analyse this, we can first look at the situation when the output is short-circuited, with the short-circuit current defined by (2). If the drivers have zero output resistance, the capacitors will charge and discharge 'instantly', and so the voltage swing ΔU will be equal to the supply voltage. The open-circuit output voltage, ignoring the diode drops, will be

$$U_{OC} = N\,U_S \qquad (4)$$

For a simple network, the short-circuit output resistance is defined by the open-circuit voltage divided by the short-circuit current, so

$$R_Z = \frac{N\tau}{C} \qquad (5)$$

However, a couple of queries now arise. Firstly, the drivers will have a finite output resistance, so the capacitor voltage swing ΔU will be less than U_S meaning that the quantity R ought to appear somewhere in the above expression.

Secondly, although a simple network can be characterised as described above, we have no evidence – as yet – that this charge-pump arrangement obeys the same rules. The output resistance could vary with voltage in an unexpected manner.

These points require a fuller analysis.

Mathematical Analysis

Output Resistance

The first point to clear up is whether the short-circuit resistance is a useful parameter. In a circuit with ideal components (and no voltage drop across the diodes), the power lost during the charge-pump operation must all be dissipated in the output resistances of the two drivers. If we can write that in terms of the load current, we can equate it to a fictitious 'output resistance', but the question is whether that resistance is also the short-circuit resistance, or whether it varies with load in some way.

Suppose the open-circuit output voltage U_{OC} is equal to NU_S and the output current is dropped across some fictitious resistance R_{out} to present a load voltage U_L. The load current is therefore

$$I_L = \frac{U_{OC} - U_L}{R_{out}} \qquad (6)$$

and the power dissipated in R_{out} is

$$\begin{aligned} P_R &= I_L{}^2 R_{out} = I_L\left(U_{OC} - U_L\right) \\ &= P_S - P_L \end{aligned} \qquad (7)$$

That equation is probably stating the obvious, but it was as well to check. It tells us that if the model features an open-circuit output voltage equal to NU_S, we can represent the power loss ($P_S - P_L$) as the output resistance R_{out} in our circuit model. The next stage is to calculate P_R and thus find a formula for R_{out}. If this turns out *not* to have any strange dependencies, it can clearly function as R_Z, as described above.

Time Constant

Consider *Figure 3b.* Here, in one half-cycle of the pump, there are ½N current paths. We have said that each path conveys the same charge, so the current must be equal in each path. ½N–1 paths involve the output resistance of both drivers (2R) and a total capacitance of ½C, whilst one path includes just R and C. We know that the current will decay exponentially, and so we can see that there are ½N paths, each with a time constant of RC, thus the overall time constant is

$$T = \tfrac{1}{2} NRC \qquad (8)$$

Figure 3a presents a similar picture.

Power Dissipation

The current sourced and sunk by the drivers can be represented by

$$I = I_0 \exp\left(-t/T\right) \qquad (9)$$

The power that is dissipated in the drivers (which have a total resistance of 2R) in a half-cycle of time ½τ is therefore

$$\begin{aligned} P_R &= \frac{4R}{\tau}\int_0^{\tau/2} dt\, I^2 \\ &= \frac{4R}{\tau} I_0{}^2 \int_0^{\tau/2} dt\, \exp^2\left(-t/T\right) \\ &= -\frac{2R}{\tau} I_0{}^2 T\left[\exp^2\left(-t/T\right)\right]_{t=0}^{\tau/2} \\ &= \frac{2R}{\tau} I_0{}^2 T\left(1 - \exp^2\left(-\tau/2T\right)\right) \end{aligned} \qquad (10)$$

In order to eliminate I_0 we write an expression for the mean source current,

$$\begin{aligned} I_S &= \frac{2}{\tau}\int_0^{\tau/2} dt\, I \\ &= \frac{2}{\tau} I_0 \int_0^{\tau/2} dt\, \exp\left(-t/T\right) \\ &= \frac{2}{\tau} I_0 T\left(1 - \exp\left(-\tau/2T\right)\right) \end{aligned} \qquad (11)$$

Writing $x = \exp(-\tau/2T)$ for brevity, and substituting (11) into (10) we obtain,

$$P_R = \frac{2R}{\tau}\left(I_S \frac{\tau}{T}\right)^2 T \frac{1 - x^2}{(1 - x)^2} \qquad (12)$$

2 A diode has a well-established formula (from theory and verified by experimentation) that describes how its forward voltage varies with current. If there truly were such a thing as a knee voltage, then it would certainly be possible to derive an equation for it. Try it – you'll find that it cannot be done: the knee voltage is nothing but a figment of your imagination.
A word of warning – be cautious of asking Google, as some web pages make rather a mess of explaining knee voltage, and they certainly would not get any marks if that were their answer to an exam question.

which reduces to

$$P_R = \tfrac{1}{2} R \frac{\tau}{T} {I_S}^2 \frac{1+x}{1-x} \qquad (13)$$

Substituting I_L for $N\,I_S$ (3), and making use of the definition of the hyperbolic cotangent (see appendix), we obtain

$$P_R = \tfrac{1}{2} N^2 R \cdot \frac{\tau}{T} \cdot {I_L}^2 \cdot \coth\left(\frac{\tau}{4T}\right) \qquad (14)$$

Output Resistance

We can now deduce that there are no special circumstances that prevent us from equating R_{out} with R_Z, and we can deduce directly from the equation that the resistance that is dissipating this power is

$$R_Z = \tfrac{1}{2} N^2 R \cdot \frac{\tau}{T} \cdot \coth\left(\frac{\tau}{4T}\right) \qquad (15)$$

or, alternatively,

$$R_Z = \frac{N\tau}{C} \cdot \coth\left(\frac{\tau}{4T}\right) \qquad (16)$$

Note that, as $R \to 0$, the coth term tends to unity, thus giving us the same answer as (5) in the earlier, simple, analysis where we did not include *R*.

This is a remarkable result as it seems to indicate that there is a power loss in an apparently finite output resistance even if the drivers themselves are 'ideal', with no output resistance. As there is no other 'real' resistive component anywhere in the circuit, what causes this power loss? I discuss this mystery in a companion article (Gibson, 2020).

To maximise the efficiency of the charge pump we need to minimise R_Z. Noting that $\coth x \approx 1/x$ when $x << 1$, we can see that if $\tau << 4T$ or, equivalently,

$$\tau << 2NCR \qquad (17)$$

the resistance is minimised at

$$R_Z \approx 2N^2 R \qquad (18)$$

In summary, the output resistance can vary from a minimum, given by (18), to a maximum, given by (5).

Observations

We can now see that an equivalent circuit model of the charge pump is a voltage source of value NU_S in series with an output resistance of value $2N^2R$. Additionally, the load current will be $1/N$ times the source current. It is interesting that this is the identical model to that which would be provided by an ideal transformer with a turns ratio of 1:*N*. The voltage goes up by *N*, the current goes down by *N* and the resistance goes up by N^2 (from $2R$ to $2N^2R$). Buoyed by that neat result, we can go on to investigate the effect of the diodes, and the effect of a driver duty cycle of less than 100%.

Duty Cycle

Suppose the drivers do not immediately switch from source to sink but have an off period in-between. If the on time is α instead of ½τ, this has the effect of modifying (15) to

$$R_Z = \tfrac{1}{2} N^2 R \cdot \frac{\tau}{T} \cdot \coth\left(\frac{\alpha}{2T}\right) \qquad (19)$$

so that, if $\tau << 4T$ we have

$$R_Z = \frac{2N^2 R}{\delta} \qquad (20)$$

where δ is the *duty cycle* defined by

$$\delta = \alpha \big/ \tfrac{1}{2}\tau \qquad (21)$$

Diode Voltage Drop

The diodes cause an additional power loss that can be added to that calculated in (14). The power loss is tedious to calculate precisely, but an exact calculation is not necessary, as we can get an estimate of the power loss by basing it on the mean current. Considering *Figure 3b* again, in a half-cycle of operation, ½*N* diodes each pass a mean current equal to $2I_L$ so, considered over a half cycle, the total diode power dissipation is $NI_LU_D(2I_L)$, where U_D is the diode forward voltage, written as a function of current.

Thus, the overall power loss due to the diodes is *as if* there was a chain of *N* diodes passing a current $2I_L$ so, without too much inaccuracy, we can describe the model as having *N* diodes in series with the output, see *Figure 4*, the difference between $U_D(I_L)$ and $U_D(2I_L)$ being small.

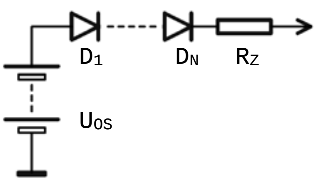

Figure 4 – Circuit Model of the PFVM

The output of a PFVM can be modelled as an ideal 1:N step-up transformer with an output of $U_{OS} = NU_S$. The power loss is represented by $R_Z = 2N^2R$ where R is the output resistance of each of the drivers, and by a chain of N diodes.

Summary of Operation

The parallel-fed voltage multiplier can be modelled as an ideal transformer, where the power losses are represented by an output resistance of N^2 times the driver impedance in series with *N* diodes. The condition for this to be a good approximation is that the cycle time $\tau << 2NRC$, see *Figure 4*. If the drivers have a duty cycle of δ (i.e. some dead time after each on period) then the output resistance increases by 1/δ.

Applications

These days, electronics has become a mature subject, with pre-designed solutions available to an unskilled designer. So, for example, there are numerous single-chip designs for switched-mode power supplies, based around inductors, which require the user to merely calculate a few simple component values. However, sometimes we might wish to look at the merits of an alternative design, and the capacitor charge pump may still have applications such as

- Bootstrapped MCU power supply
- Flashtube HT generator
- LED cap-lamp
- Bio-electronics, e.g. TENS
- Low-power environmental sensors

A cap-lamp design is interesting because there are a couple of features of the charge-pump that lend it to such use. Firstly, the output load capacitor is not needed if the load is an LED, because the LED will clamp the output at a more-or-less constant voltage that allows the circuit to function properly. Secondly, we can control the LED current simply by altering the duty-cycle, which alters the effective output resistance (current feedback is still needed, of course). The LED will receive a pulsed current because current is only delivered in one of the half-cycles of operation, but we could use two charge pumps 'back to back' to make a 'full-wave' pump. See (Kim, Cho and Hong, 2008).

Concluding Remarks

A charge pump is interesting because it is simpler, and more tolerant of design errors *and* building errors than an inductor-based design. I have shown that it has a surprisingly simple circuit model too. It is probably electrically quiet, which might be useful if it is operating near radio equipment. Understanding its operation opens up some interesting design avenues, e.g. a high-power voltage to current converter for cap-lamps. Above all, it is *different* and, with fewer opportunities for electronics enthusiasts to do something novel and unusual, it would seem to be an interesting design to experiment with.

You may think that the circuit model of *Figure 4* is 'obvious' and that this analysis has been rather a waste of time – if the output voltage goes up by *N* then 'of

continued on page 22

We can also ask why this phenomenon does not appear to apply to *all* uses of capacitors. After all, smoothing capacitors are used in power supplies and, if the 50% rule applied, it would significantly reduce their efficiency. This must now be analysed.

Partially Charging a Capacitor

Assume that a capacitor's voltage is raised by a small amount ΔU in time Δt, so

$$\left.\begin{array}{l}\Delta E_B = U_B I\cdot\Delta t\\ \Delta E_R = I^2 R\cdot\Delta t\end{array}\right\} \Rightarrow \frac{\Delta E_R}{\Delta E_B} = \frac{I\,R}{U_B} = 1-\frac{U_C}{U_B} \qquad (6)$$

and so the efficiency of charging, η, is

$$\eta = \frac{\Delta E_C}{\Delta E_B} = 1-\frac{\Delta E_R}{\Delta E_B} = \frac{U_C}{U_B} \qquad (7)$$

The efficiency seems to depend on having the capacitor and the supply voltage close in potential. This general principle crops us in physics quite often – for example the most efficient thermal energy flow occurs between bodies that are close in temperature. Obviously, there is a caveat, which is that the *rate* of flow of energy is lower if the potentials are closer.

We can extend this argument to determine the overall efficiency when we charge the capacitor between U_{C1} and U_{C2}. Suppose it charges from empty, with

$$I = I_0 \exp(-t/T) \qquad (8)$$

The energy drawn from the supply and the energy dissipated in the charging resistor during a period from τ_1 to τ_2 are

$$\left.\begin{array}{l}E_R = I_0{}^2 R\cdot\int_{\tau_1}^{\tau_2} dt\exp^2(-t/T)\\ E_B = U_B I_0\cdot\int_{\tau_1}^{\tau_2} dt\exp(-t/T)\end{array}\right\} \qquad (9)$$

so the ratio of the two is

$$\begin{aligned}\Rightarrow \frac{E_R}{E_B} &= \tfrac{1}{2}\frac{I_0 R}{U_B}\cdot\frac{\left[\exp^2(-t/T)\right]_{\tau_1}^{\tau_2}}{\left[\exp(-t/T)\right]_{\tau_1}^{\tau_2}}\\ &= \tfrac{1}{2}\frac{I_0 R}{U_B}\cdot\left(\exp(-\tau_1/T)+\exp(-\tau_2/T)\right)\\ &= \tfrac{1}{2}\frac{1}{U_B}\left(U_B-U_{C1}+U_B-U_{C2}\right)\\ &= 1-\tfrac{1}{2}\frac{U_{C1}+U_{C2}}{U_B}\end{aligned} \qquad (10)$$

Thus the efficiency – the amount of energy in the capacitor as a function of the energy taken from the supply – is

$$\frac{E_C}{E_B} = 1-\frac{E_R}{E_B} = \tfrac{1}{2}\frac{U_{C1}+U_{C2}}{U_B} \qquad (11)$$

Although it would take an infinite time to charge the capacitor all the way to U_B, we can get fairly close in a reasonably finite time, so we can simplify this expression by assuming $U_{C2}\approx U_B$ and then, writing $\Delta U = U_B - U_{C1}$, we obtain

$$\frac{E_C}{E_B} = 1-\tfrac{1}{2}\frac{\Delta U}{U_B} \qquad (12)$$

This shows that the efficiency of power transfer is 50% if we charge the capacitor from empty ($U_C = 0$, i.e. $\Delta U = U_B$), rising to close to 100% if the starting voltage is close to the supply voltage ($\Delta U \to 0$).

When considered in terms of the ripple voltage on a power supply smoothing capacitor, this tells us that 10% ripple voltage gives a 95% power efficiency, and that 2% ripple gives a 99% efficiency. Smoothing capacitors are usually specified with a low 'equivalent series resistance', (ESR) so that the ripple current does not cause too much power dissipation. The ESR is a combination of ohmic resistance and dielectric losses. However, we can now see that the situation is not quite as simple as that – because even if the capacitor has a zero ESR, there is still some 'lost' power that does not depend on there being any dielectric loss.

If the smoothing capacitor has no losses, where does the missing energy manifest itself?

Concluding Remarks

I have shown that when capacitors charge and discharge, the resulting flow of charge causes energy to be dissipated in the charging resistor. The conundrum is that if the resistor has a value of zero, energy appears to vanish. Clearly, it does not actually do so or 'somebody would have noticed by now'. As engineers, it is not essential that we address this problem, because we will always a have a finite resistance in the circuit. Nevertheless, by considering the problem, and examining it in different ways, we can get a valuable insight into the physics of the situation, on which the reader is invited to dwell.

References

Catt, I; Walton D. & Davidson M. (1979), Digital Hardware Design [Chapter 5: Component Pulse Response]. London: Macmillan Press. ISBN 0-333-25981-5. Available online at **ivorcatt.org/digital-hardware-design.htm**, accessed 7 June 2020.

Gibson, David (2020), *The Parallel-Fed Voltage Muliplier*, CREGJ **111**. pp10-12, Sept. 2020.

The Parallel-Fed Voltage Multiplier

Continued from page 12…

course' the input current goes down by N and 'of course' the driver resistance is multiplied by N^2. But to state those assumptions as facts, before they are proven, would be wrong; and how would you explain the existence of the coth term?

The salient point is that charging a capacitor can be an inefficient operation (Gibson, 2020). Consider that when the driver resistance is zero, the circuit *still* has a finite output resistance, given by (5); and that $R_Z = 2N^2R$ is merely an approximation, true only when $\tau << 4T$. I, myself, was astonished that the circuit model turned out to be so simple; and that we can control the output resistance merely by altering the duty cycle.

Appendix: Hyperbolic Cotangent

The hyperbolic cotangent function, coth x, is defined in terms of the hyperbolic sine and cosine. These, in turn are defined in terms of the exponential function, as

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{\exp x + \exp(-x)}{\exp x - \exp(-x)} \qquad (1)$$

There are some equivalent forms, found by multiplying through by $\exp(x)$ or $\exp(-x)$.

$$\coth x = \frac{\exp 2x + 1}{\exp 2x - 1} = \frac{1+\exp(-2x)}{1-\exp(-2x)} \qquad (2)$$

The coth function is characterised by

As $x \to 0$, $\coth x \to \infty$
As $x \to \infty$, $\coth x \to 1$
For $x << 1$, $\coth x \approx 1/x$

Some Symbols Used

I_0	Peak source current in each half cycle
I_L	Load current
I_S	Source current
N	Number of stages in charge pump
R	Output resistance of driver
R_L	Load resistance
R_{out}	Effective output resistance
R_Z	Short circuit output resistance
τ	Period of driver operation
T	Time constant of RC network
U_D	Diode forward voltage
U_L	Load voltage
U_{OC}	Open circuit output voltage
U_S	Source voltage

References

Gibson, David (2020), *The Mystery of the Missing Energy*, CREGJ **111**. pp20-22, September 2020.

KIM Shiho, Jung-Hyun CHO, Suk-Kyung HONG, 2008, *A Full-Wave Voltage Multiplier for RFID Transponders*, IEICE Transactions on Communications, **E91–B**, (1), pp388-391. January 2008.

Purves R.D. and Prescott C, 1984, *The Parallel-Fed Voltage Multiplier*, Wireless World, **90**(1578), p43 & 45. March 1984. Available at **worldradiohistory.com/UK/Wireless-World/80s/Wireless-World-1984-03.pdf**, accessed 9 Aug 2020 (file size 11MB).